Journal of Statistics: Advances in Theory and Applications
Volume 6, Number 1/2, 2011, Pages 19-54

ROBUST MULTIPERIOD POVERTY COMPARISONS

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Abstract

Building on the literature on multidimensional poverty, we propose stochastic dominance tests for multiperiod poverty that allow poverty orderings to be established over time and space that are robust to the chosen poverty measure, poverty line, and aggregation procedure. These tests imply the creation of dominance surfaces for different time spans and testing for significant differences. We elaborate the method first for the bi-dimensional case, using as the dimensions income observed over two periods: One at the beginning and another at the end of a time span. Subsequently, we extend it to the case, where incomes are observed over n periods. We illustrate our approach by

2010 Mathematics Subject Classification: I32, C18, D63.

Keywords and phrases: multiperiod poverty, stochastic dominance, poverty dynamics, chronic poverty.

Received October 12, 2011

performing poverty comparisons by using data for Indonesia and Peru. The discussion is embedded in the literature on chronic poverty.

1. Introduction

The literature dealing with the measurement of poverty has seen a significant development during the past three decades. This has been accompanied by an increasing availability of more and better household level data, in particular longitudinal data, i.e., data for the same observation units over time. Originally, the literature mainly focused on poverty at a given point in time (Sen [33], Foster et al. [18], Atkinson [1]). The study of poverty changes was limited to the comparison of poverty measurements computed for repeated cross-sections. Such comparisons rely on the anonymity axiom and do not allow any statements to be made about the mobility of people across the income distribution over time.

With the availability of panel data, it was possible to relax the anonymity axiom and also to analyze 'poverty dynamics', i.e., transitions from 'poor' to 'non-poor' and vice versa (Atkinson et al. [2], Grootaert and Kanbur [23], Fields and Ok [17], Maasoumi and Zandvakili [31]). From this, a literature then emerged distinguishing chronic from transitory poverty (see, for instance, Rodgers and Rodgers [32], Jalan and Ravallion [29], Hulme and McKay [28], Duclos et al. [14]). In this literature, chronic poverty usually refers to measuring in one way or another the share of individuals, who find themselves in poverty over a longer period of time. It also involves analyzing the determinants of the probability of remaining in poverty. In contrast, transitory poverty refers to short spells of poverty interrupted by spells beyond the poverty threshold. Within that literature, two strands can be distinguished: One that defines chronic poverty through the number of consecutive spells in poverty – the 'spells approach' (see Bane and Ellwood [3], Stevens [35], Hulme and McKay [28]) – and one that decomposes changes in total poverty into a component due to chronic poverty and a component due to transitory poverty – the 'components approach' (for instance, Jalan and Ravallion [29]). Both approaches typically require the use of strong assumptions, in

particular, an agreement on the specific poverty line and on the specific poverty function. This makes the results that are derived from such approaches somehow arbitrary or at least very much dependant on these assumptions. Foster [22], for instance, suggested a new family of chronic poverty measures based on the well-known Foster-Greer-Thorbecke snap-shot poverty measures (Foster et al. [18]). He identifies the chronically poor by using two cutoffs: A standard poverty line, which identifies the time periods during which a person is poor, and a duration cutoff, which is the minimum percentage of time a person must be in poverty in order to be chronically poor. All measures within that class satisfy a series of general axioms for chronic poverty measures and an additional set of axioms that are applicable to measures based on a spells approach to chronic poverty. Foster applies a very simple treatment of time; every period is given the same weight.

Other authors have started to investigate such underlying implications in more depth and worked out the theoretical foundations of such measurements. Cruces [12], for instance, has developed concepts to aggregate incomes over multiple periods, i.e., trajectories of income over time using an evaluation function that explicitly captures the risk aversion of households. While such an approach has the advantage of accounting for the negative effects of income variability on the well-being of households, it requires an 'arbitrary' assumption to be made about how exactly 'risk-adjusted mean income' is best computed. Calvo and Dercon [9] also discuss various measures of intertemporal poverty. They develop an axiomatic approach to the measurement of poverty over multiple spells of time. As in Cruces [12], they deal with various problems such as giving weight to different spells of poverty and discuss whether a higher income in the future can compensate for poverty today. One of their propositions is to reject the notion of time discounting, which is standard in most of the economics literature, and to adopt instead the principle of 'universalism', which argues strongly for valuing distress equally and independently of the time period in which the poverty occurred. Calvo and Dercon [9] illustrate their concept with Ethiopian panel data.

All these papers, starting with Foster [22], are based on axioms; however, they do not elaborate the full characterization of the measures that would satisfy these axioms. This is in turn done by Hoy and Zheng [25] (see also Hoy et al. [26]) and Bossert et al. [6]. Hoy and Zheng [25] propose that lifetime poverty should be influenced by both the poverty status in each single period and the level of 'permanent' lifetime consumption. Moreover, the distribution of poverty spells over the lifetime should be accounted for. Based on the axioms, Hoy and Zheng [25] characterize classes of lifetime poverty indices along with dominance conditions of poverty orderings on the individual and societal level. Bossert et al. [6] also propose a lifetime poverty measure, but they explicitly deviate from several assumptions made in Hoy and Zheng [25]. They allow the poverty line to vary over time and even allow for relative poverty lines, which involves abandoning the path independence considered by Hoy and Zheng [25]. They also allow comparisons, where the length of spells to be compared is not identical. Finally, the paper is very explicit regarding how persistence should be taken into account. As in Hoy et al. [26], a higher level of lifetime poverty is assigned to situations, where poverty occurs earlier in life and where poverty is experienced in consecutive rather than separated periods. These two papers, as well as those mentioned before, have greatly contributed to the literature on poverty and time.

In this paper, we take a step back, i.e., we make fewer assumptions on how multiperiod poverty should be measured. Instead, we suggest applying the stochastic dominance methodology when comparing poverty over multiple periods across time and space. Intertemporal poverty orderings based on stochastic dominance are robust to a large set of poverty measures and poverty lines. As we will show, robustness to the selection of both the poverty line and the poverty measure also includes robustness to period specific weighting. This is an important feature, which is particularly useful in cross-group or cross-country comparisons, where we may want the timing of poverty in life to matter (as in Hoy et al. [26], Bossert et al. [6]).

We consider two sets of spells, either across time for a given group of individuals or across space for different groups. Throughout this paper, we borrow and adapt the concept of multidimensional poverty orderings (Duclos et al. [15, 16]) to compare multiple-period poverty over time and space.[1] Instead of multiple dimensions, such as income, education, and health, we consider multiple periods, which raises a couple of methodological issues. We also extend our approach to include the concept of relative poverty. However, we do not address issues related to income uncertainty and the disutility (utility reductions) induced by income volatility.

The remainder of our paper is organized as follows. In Section 2, we present our methodology. We discuss our approach first for the case, where incomes are observed over a single period of time. This is helpful, since the one-period concept can be easily transferred to the multiperiod case. We then extend the approach to the case, where incomes are observed over two periods. Finally, we extend it to the case, where incomes are observed over n-periods. In Section 3, we implement our methodology empirically and analyze multiperiod poverty in Indonesia and Peru. In Section 4, we discuss our results and conclude.

2. Methodology

2.1. Stochastic dominance in a one-period welfare measure

In this section, we briefly introduce the basic concept of stochastic dominance and show how it can be applied to establish poverty orderings in the one-period case.[2]

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[1] See also Duclos et al. [16] and the seminal papers by Bourguignon and Chakravarty [7, 8].

[2] For a comprehensive overview on stochastic dominance applied to the analysis of income distributions, see, for instance, Barrett and Donald [4], Davidson and Duclos [13], Jenkins and Lambert [30].

We assume that individual well-being, λ, is a function of y, a well-being indicator, for instance, income received in period t. Let y be defined over the interval $[0, \infty]$, where the set of distributions of well-being indicators is $\Psi := F : [0, \infty] \to [0, 1]$. We assume a non-decreasing well-being function without imposing anything concerning the exact contribution of y to well-being, $\lambda(y)$, where $\partial\lambda(y) / \partial(y) \geq 0$. An individual is assumed to be poor if well-being $\lambda(y)$ is below a poverty frontier, $\lambda(z)$, where z is the poverty line belonging to the well-being indicator. The poverty set can then be defined as $\Lambda(\lambda) = \{y|\lambda(y) \leq \lambda(z)\}$, with $\lambda(z) = 0$.

We consider, following Atkinson [1], all additively separable poverty measures P that are non-decreasing in $\lambda(y)$ and anonymous. We denote this set of poverty measures as Ξ_1. Our poverty measure can be computed by [3]

$$P(F; \lambda) = \int_{\Lambda(\lambda(z))} p(\lambda(y), \lambda(z)) dF(y). \tag{1}$$

Our set of poverty measures, Ξ_1, includes, for instance, the Watts measure of poverty (Watts [37]), where $p(y, z) = (\ln z - \ln y)$, and all poverty measures within the Foster-Greer-Thorbecke family, P_α (Foster et al. [18]) with $\alpha \geq 0$ (Foster and Shorrocks [19, 20]), where $p(y, z) = (1 - y / z)^\alpha$.[4] If well-being is only measured along one dimension, the one-period stochastic dominance curve is given by

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[3] Note: In the one period case, Equation (1) can be rewritten as $P(F; z) = \int_0^z p(y, z)$ $dF(y)$.

[4] The Foster-Greer-Thorbecke poverty measure has the formula $P_\alpha = 1 / N \sum_{i=1}^{n}$ $(1 - \frac{y_i}{z})^\alpha$, where N is the total number of individuals $i = 1, \ldots, N$. The parameter $\alpha > 0$ is a poverty aversion parameter: $\alpha = 0$ yields the poverty headcount index, $\alpha = 1$, the poverty gap index, and $\alpha = 2$, the poverty severity index (Foster et al. [18]).

$$P^{\alpha}(F;\, z) = \int_0^z (z - y)^{\alpha}\, dF(y), \tag{2}$$

for $\alpha \geq 0$.

Tests of stochastic dominance are today widely used to establish poverty orderings $\mathbf{D}$ that are robust for a broad class of poverty measures, $P(F;\, z)$, and a large range of poverty lines, $z \in [0, \infty]$. Given two distributions $F \in \Psi$ and $G \in \Psi$, the first order stochastic dominance condition (FSD), $\mathbf{D_1}$, states

$$F\mathbf{D_1}G \;\forall\, P \in \Xi_1,\; z \in [0,\, z^{\max}]$$

$$\Leftrightarrow\; P^0(F;\, z) - P^0(G;\, z) < 0 \;\forall\, z \in [0,\, z^{\max}], \tag{3}$$

where $F\mathbf{D_1}G$ means that F has unambiguously less poverty than G with respect to all poverty indices belonging to the class Ξ_1 and all poverty lines within the range $[0,\, z^{\max}]$.

For first order stochastic dominance (FSD) orderings, it is sufficient to compare the dominance curve of the well-being indicator in period 1, $P^0(F;\, z)$, with its analogue in period 2, $P^0(G;\, z)$. If FSD does not hold, higher-order stochastic dominance tests can be applied to generate robust poverty orderings. Higher-order dominance requires further assumptions to be added on how the function $p(y,\, z)$ evolves with y. For instance, second order stochastic dominance (SSD) requires $p(y,\, z)$ to be specified in such a way that P satisfies the Pigou-Dalton transfer principle (see, e.g., Foster and Jin [21] and Shorrocks and Foster [34]). The Pigou-Dalton transfer principle states that a transfer of income from a richer to a poorer person will not increase poverty as long as that transfer does not

reverse the ranking of the two. If we denote the set of all Daltonian[5] poverty measures, Ξ_2, then SSD, $\mathbf{D_2}$, states

$$F\mathbf{D_2}G \;\forall\, P \in \Xi_2,\; z \in [0,\, z^{\max}]$$

$$\Leftrightarrow\; P^1(F;\, z) - P^1(G;\, z) < 0 \;\forall\, z \in [0,\, z^{\max}], \tag{4}$$

where $F\mathbf{D_2}G$ means that F has unambiguously less poverty than G with respect to all poverty indices belonging to the class Ξ_2 and all poverty lines within the range $[0,\, z^{\max}]$. If second order dominance also does not hold, it is possible to test for third order dominance. This would of course further limit the set of applicable poverty measures by imposing even more restrictive axioms. We restrict our analysis to FSD and SSD.[6]

2.2. Stochastic dominance in a two-period welfare measure

We now extend the concept outlined above by focusing on welfare measures that are defined over two periods instead of one period. The time between the beginning of the first period and the end of the second period is called '*time span*'. Such time spans are considered in the chronic poverty literature. As we outlined in the Introduction, the measures of chronic poverty, or, more generally, intertemporal or multiperiod poverty, that are usually proposed either count the periods (or 'spells') during which an individual was below the poverty line, or look at some form of

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[5] The Daltonian class of poverty measures contains, for instance, the Clark et al. [11] measure, the Watts [37] measure, and the Chakravarty [10] measure (see Zheng [39]).

[6] For a discussion of poverty orderings beyond second degree stochastic dominance, see, for instance, Zheng [38]. It should also be noted that we do not consider weak stochastic dominance, because statistically, it is impossible to distinguish weak and strong stochastic dominance. For strict stochastic dominance, $P^{\alpha}(F;\, z) - P^{\alpha}(G;\, z) < 0$ is needed for all poverty measures to be greater in z, while $P^{\alpha}(F;\, z) - P^{\alpha}(G;\, z) \le 0$ is sufficient for weak stochastic dominance. Thus, strict stochastic dominance, as defined in Equation (3), implies weak stochastic dominance.

mean income over these periods and compare this mean income to the poverty line. Although these measures are very straightforward and lead to unambiguous orderings, this comes at a cost: The results typically depend, as argued above, on the chosen poverty line and poverty measure. In this paper, we take a step back and explore the possibility of applying stochastic dominance tests to compare intertemporal poverty over time and space. These tests imply verifying that for a large set of poverty lines and poverty measures intertemporal poverty during one time span was lower or higher than during another time span. These time spans can either be observed for a given group at different points in time or for two different groups at the same time.

To apply stochastic dominance tests to intertemporal poverty, we employ the methodological framework that was introduced by Duclos et al. [15, 16] to undertake stochastic dominance comparisons for poverty measures defined at a single point in time, but along two or more dimensions, such as health and education. Their robustness tests refer to the poverty measure and the poverty line, and, thereby, implicitly to the way, both dimensions are aggregated. In our case, dimensions refer to different periods that constitute a time span.

We start with first-order stochastic dominance (FSD), but will show, in case FSD does not hold, how higher order tests can be applied to come to robust orderings. We then extend the analysis to the n-period case, i.e., to comparisons, where time spans constitute more than two periods. Throughout the analysis, we focus in particular on those problems that are specific to the comparison of time spans across time or groups.

It is important to note that, whereas in Duclos et al. [15, 16], each dimension is assessed against its own specific poverty line (i.e., the health poverty line and the education poverty line), in our case, poverty over each time span is measured with the same type of poverty line (i.e., referring to the same dimension). However, in the most general case, this poverty line may vary in terms of its magnitude across periods. This may

become relevant, if, for instance, we want to give more weight to income episodes earlier in life than those later in life. Given that the exact weights to apply must somehow depend on an arbitrary choice, it is interesting to test the robustness of the poverty ordering with respect to the chosen weights.

Our two-period well-being function has as its arguments y_1 and y_2, i.e., income received in period 1 and income received in period 2. Hence, the well-being function can then be written as $\lambda(y_1, y_2) : \Re^2 \rightarrow \Re | \partial\lambda(y_1, y_2) / \partial(y_1) \geq 0, \partial\lambda(y_1, y_2) / \partial(y_2) \geq 0$. We define an individual to be poor, if her overall well-being $\lambda(y_1, y_2)$ is below the unknown poverty frontier. In the two-period case, the poverty frontier may either be a single point, z, if $z_1 = z_2$, or a locus of points, if $z_1 \neq z_2$. The poverty locus is defined as $\lambda(y_1, y_2) = 0$. It should be noted that the locus includes $z_1 = z_2$. The overall set of poor people is then defined as $\Lambda(\lambda) = \{y_1, y_2 | \lambda(y_1, y_2) \leq 0\}$.

We consider all additively separable, non-decreasing, and anonymous poverty measures P, i.e., we impose anonymity when comparing time spans (across time or space), but within time spans anonymity is obviously removed. Additionally, we require y_1 and y_2 to be ALEP substitutes in $\lambda(y_1, y_2)$.[7] This assumption implies that an increase of the well-being indicator in one period leads to a greater increase in overall well-being the lower the well-being indicator in the other period. Hence, our concept of multiperiod poverty accounts for the correlation between individuals' outcomes across both periods (i.e., within time spans). We denote this set of poverty measures as $\Xi_{1,1}$. Transferring Equation (1) to the two-period case, the poverty measure reads

[7] 'ALEP' means substitution in the sense of Auspitz, Lieben, Edgeworth and Pareto (ALEP), i.e., we impose: $\frac{\partial\lambda^2(y_1, y_2)}{\partial y_1 \partial y_2} \geq 0, \forall\ y_1, y_2$.

$$P(F;\,\lambda) = \iint_{\Lambda(\lambda(z_1, z_2))} p(\lambda(y_1,\, y_2),\, \lambda(z_1,\, z_2))dF(y_1,\, y_2). \tag{5}$$

Accordingly, the two-period stochastic dominance surface is given by

$$P^{\alpha_{y1}\alpha_{y2}}(F;\, z_1,\, z_2) = \int_0^{z_1}\int_0^{z_2}(z_1 - y_1)^{\alpha_{y1}}(z_2 - y_2)^{\alpha_{y2}}\, dF(y_1,\, y_2), \tag{6}$$

for $\alpha \geq 0$. A comparison of two time spans is denoted as $T_a = [t_{1a};\, t_{2a}]$ vs. $T_b = [t_{1b};\, t_{2b}]$, where t now has an index for the period within each time span, year 1 or year 2, and an index for the time span, time span a or time span b.

Given two distributions, $F(y_{1a},\, y_{2a}) \in \Psi$ and $G(y_{1b},\, y_{2b}) \in \Psi$, the first order stochastic dominance condition, $\mathbf{D_{1,\,1}}$, states

$$F\mathbf{D_{1,\,1}}G \;\forall\, P \in \Xi_{1,1},\, z_1 \in [0,\, z_1^{\max}],\, z_2 \in [0,\, z_2^{\max}]$$

$$\Leftrightarrow P^{0,0}(F;\, z_1,\, z_2) - P^{0,0}(G;\, z_1,\, z_2) < 0 \;\forall\, z_1 \in [0,\, z_1^{\max}],\, z_2 \in [0,\, z_2^{\max}], \tag{7}$$

where $F\mathbf{D_{1,\,1}}G$ means that multiperiod poverty is lower over time span T_a than over time span T_b with respect to all poverty indices belonging to the class $\Xi_{1,1}$ and all poverty lines within the range $[0,\, z_1^{\max}]$ and $[0,\, z_2^{\max}]$.[8]

As in the one-period case, tests of higher order dominance could be equally well established by imposing further assumptions regarding the effect of y on $p(y_1,\, y_2,\, z_1,\, z_2)$. For instance, holding constant the distribution in period 2, we could impose that a transfer from a richer to a poorer person in period 1 reduces poverty. Symmetrically, we would then impose the same transfer sensitivity on period 2.

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[8] Note that z_{1a} cannot be different from z_{1b}.

Depending on the specific definition of the locus of the poverty frontier, multiperiod poverty comparisons can be performed according to the 'intersection' and the 'union' poverty definition (Duclos et al. [15]). Intersection poverty means in our case that someone is considered poor, if well-being is below the poverty threshold in *both* periods. The concept of 'intersection' multiperiod poverty is therefore closely related to the concept of chronic poverty (see, e.g., Hulme and Shepard [27]). As outlined above, chronic poverty is usually measured according to the consecutive number of spells below the poverty line or a certain average income below the poverty line. Intersection poverty captures this idea. Intersection poverty is illustrated in Figure 1 by the crosshatched area under the function $\lambda_1(y_1, y_2)$ (dashed line). Union poverty means that a person is considered poor, if well-being is below the poverty threshold in one of the two periods; thus, this concept has similarities with the concept of transient poverty. This is represented in Figure 1 by the entire shaded area under the function $\lambda_2(y_1, y_2)$ (dotted line). In the empirical part of our paper, we emphasize the parallels with the concept of chronic poverty and therefore focus on intersection poverty.

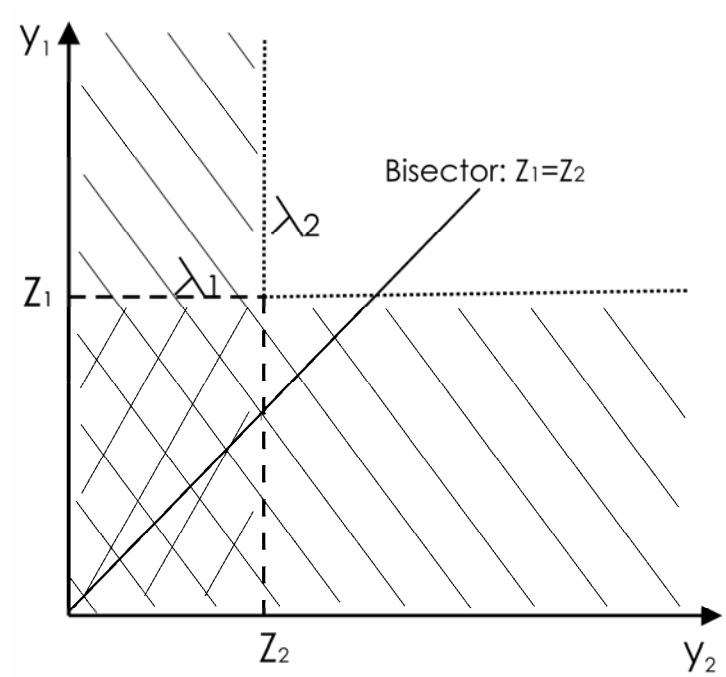

Figure 1. Test domain for dynamic poverty comparisons.

However, for multiperiod poverty comparisons, Equation (5) holds according to the intersection as well as according to the union definition of poverty, depending on the locus of $\Lambda(\lambda)$. Focusing on intersection poverty, Equation (5) can be rewritten as $P(F;\, z_1,\, z_2) = \int_0^{z_1}\int_0^{z_2} p(y_1,\, y_2,\, z_1,\, z_2)dF(y_1,\, y_2)$.

As before, the specific definition of the locus of the poverty frontier is key to the stochastic dominance analysis. When multiperiod stochastic dominance is used, it is possible and often reasonable to use the same poverty lines across dimensions, i.e., periods. Again, in this case, the locus of the poverty frontier is reduced to a single point, $z,\, z_{t1} = z_{t2}$. Poverty orderings are then robust to a broad class of poverty measures and a large set of poverty lines. However, since poverty lines (or the weights)

would be fixed over time, i.e., across stochastic dominance dimensions, robustness to different weighting schemes across periods is not relevant anymore. In this case, Equation (5) becomes

$$P(F;\, \lambda) = \iint_{\Lambda(\lambda(z,z))} p(\lambda(y_1,\, y_2),\, \lambda(z,\, z))dF(y_1,\, y_2). \tag{8}$$

Obviously, any analysis based on the more general Equation (5) includes all results based on Equation (8). Limiting the analysis to the bisector of the dominance surfaces, as is essentially carried out in Equation (8), particularly makes sense, if intertemporal poverty comparisons over time are undertaken. Using Equation (8), the dominance criteria simplifies in this case to

$$F\mathbf{D}_{\mathbf{1},\,\mathbf{1}}G \;\forall\, P \in \Xi_{1,1},\; z \in [0,\, z^{\max}]$$

$$\Leftrightarrow\; P^{0,0}(F;\, z,\, z) - P^{0,0}(G;\, z,\, z) < 0 \;\forall\, z \in [0,\, z^{\max}]. \tag{9}$$

We now show why our method also implies robustness to the weighting scheme. Again, this is mainly relevant for comparisons in intertemporal poverty across groups. Robustness to the weighting scheme is implicit, because changing the weights is equivalent to altering the poverty lines within time spans, because varying the poverty lines means adjusting the income necessary to pass the poverty frontier. Finally, this is also equivalent to the use of a discount rate. The reasoning is exactly the same as in Duclos et al. [15, 16]. In their context of different well-being dimensions (not periods), robustness to the poverty line and the poverty measure implies robustness to the way in which, well-being along multiple dimensions is aggregated to one aggregate indicator of well-being. For illustrative purposes, Duclos et al. [16] discuss the United Nations' 'Human Development Index' (HDI), which is a composite index consisting of an income, education, and life expectancy index. Robustness to dimension-specific poverty lines is, in this example, equivalent to robustness to the weight that is given to each single component of the HDI.

In our case, the weighting refers to the relative importance of income across time. If, for instance, as postulated Hoy et al. [26] and Bossert et al. [6], income experienced earlier in life matters more, we would place a greater weight on period-one income within both time spans than on period-two income. In practice, and if a poverty measure of the type discussed above is the measure of interest, such reweighting can be carried out simply by applying different poverty lines to period 1 and 2 within both time spans. Given that such weights, or discount rates, are a rather arbitrary choice, we show how the robustness of intertemporal poverty comparison with respect to weighting can be established. This is, again, relevant, in particular, for intertemporal poverty comparisons across groups.

The following simple example can illustrate this issue. Assume intertemporal poverty is to be compared between two countries, A and B. There are two individuals living in each country. Incomes observed for period 1 and period 2 are given by (0.25, 1.5) for the first individual of country A, I_{A1}, and by (0.75, 1.75) for the second individual, I_{A2}. For the first individual of country B, I_{B1}, the income trajectory is (0.75, 1) and for the second individual, I_{B2}, the income trajectory is (1.0, 1.5). In this example, we rely on the poverty gap index, $P_1 = \frac{1}{N}\sum_{i=1}^{n}(\frac{1-y_i}{z})$.

In the first case, the poverty line is fixed at 3 across countries and over time. The weight attributed to each period is $w_{t1} = w_{t2} = 1$. In this case, the poverty profiles do not differ, since $P_{1A} = P_{1B}$

$$P_{1A} = \frac{1}{4}[(1-\frac{0.25}{3}) + (1-\frac{1.5}{3}) + (1-\frac{0.75}{3}) + (1-\frac{1.75}{3})] = \frac{31}{48},$$

$$P_{1B} = \frac{1}{4}[(1-\frac{0.75}{3}) + (1-\frac{1}{3}) + (1-\frac{1}{3}) + (1-\frac{1.5}{3})] = \frac{31}{48}.$$

Now suppose, we assume that income experienced earlier in life matters more. We would give more weight to period 1, say $w_{t1} = 1.25$,

than to period 2, say $w_{t2} = 1$. The poverty line is still fixed at 3 across countries and over time. The relevant profiles are

$$P_{1A} = \frac{1}{4}[(1 - \frac{1.25 * 0.25}{3}) + (1 - \frac{1.5}{3}) + (1 - \frac{1.25 * 0.75}{3}) + (1 - \frac{1.75}{3})] = \frac{40}{64},$$

$$P_{1B} = \frac{1}{4}[(1 - \frac{1.25 * 0.75}{3}) + (1 - \frac{1}{3}) + (1 - \frac{1.25 * 1}{3}) + (1 - \frac{1.5}{3})] = \frac{39}{64}.$$

Since incomes in the first period are lower on average in country *A*, intertemporal poverty is higher in *A* than in *B*.

Now, suppose the poverty line in period 2 is held at 3, while the poverty line in period 1 is lowered to give more weight to the first period of a time span. As demonstrated below, a poverty line of 2.4 in year 1 gives exactly the same result as attributing a weight of 1.25 to period 1

$$P_{1A} = \frac{1}{4}[(1 - \frac{0.25}{2.4}) + (1 - \frac{1.5}{3}) + (1 - \frac{0.75}{2.4}) + (1 - \frac{1.75}{3})] = \frac{40}{64},$$

$$P_{1B} = \frac{1}{4}[(1 - \frac{0.75}{2.4}) + (1 - \frac{1}{3}) + (1 - \frac{1}{2.4}) + (1 - \frac{1.5}{3})] = \frac{39}{64}.$$

2.3. Relative poverty comparison

So far we have proposed the methodology of multiperiod poverty comparisons for the concept of absolute poverty, which considers an absolute poverty frontier and keeps track of people, who either stay below or cross this fixed frontier. However, the methodology of multiperiod poverty comparisons is equally well applicable to the concept of relative poverty. Relative poverty also keeps track of people, who either stay below or cross the poverty line, but the frontier becomes endogenous; for example, it is often expressed as a ratio of the median income. In terms of the chronic poverty literature, comparisons of multiperiod relative poverty imply to look at changes in chronic relative poverty, where chronic relative poverty means to be persistently relatively poorer than the median person. The concept of 'chronic relative poverty' has to our knowledge only rarely been used in the chronic poverty literature, but it

has some similarities with the concept of 'social exclusion' as formulated by Bossert et al. [5]. Hulme and Shepard [27] note: "[...] while it is possible to assess chronic poverty in either absolute or relative terms, most existing work [...] focuses on chronic absolute poverty. Such a focus is consistent with the approach of most poverty analysis in developing countries. But, [...] chronic relative poverty (i.e., always being in the bottom quintile of a country's income distribution) may be as hard, or even harder to escape than chronic absolute poverty." (p. 405).

To test for differences in relative poverty between two time spans, we standardize household expenditures by a relative poverty line $\tilde{z}$, i.e., $\tilde{y} = y / \tilde{z}$. We choose $\tilde{z} = 50\%$ of median income.[9] Accordingly, a relative income of 1, for example, means that the individual's income is exactly half of the median income.

Transferring Equation (5) to the concept of relative multiperiod poverty, the poverty measure reads

$$P(F;\, \lambda) = \iint_{\Lambda(\lambda(\tilde{z}_1, \tilde{z}_2))} p(\lambda(\tilde{y}_1,\, \tilde{y}_2),\, \lambda(\tilde{z}_1,\, \tilde{z}_2))dF(\tilde{y}_1,\, \tilde{y}_2), \qquad (10)$$

where $\tilde{y}$ is standardized household income and $\tilde{z}$ is the relative poverty line.

2.4. Stochastic dominance in a *n*-period welfare measure

Obviously, poverty comparisons across two time spans demand panel data over at least two rounds per time span.[10] An obvious question, which arises is how to construct time spans, if we have more than two rounds of panel data, which then also cover a longer period. For instance, we may want to compare chronic poverty over a longer period of time in country A

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[9] Note that it does not matter which share of the median is used as the poverty line.

[10] Note again that we postulate non-anonymity within time spans, but anonymity across time spans.

and country B using n rounds of panel data for each country for this purpose. Or, in the case of an intertemporal comparison, we may have multiple rounds of panel data to investigate the dynamics of chronic poverty. In the n-period case, our well-being measure becomes $\lambda(y_1, y_2, \ldots, y_n)$, while the poverty locus becomes an n-dimensional space.

So, the question is: How should time spans be constructed in that case? How many periods should constitute a time span? In the intertemporal case, which period should be the end of the first and the beginning of the second time span? Depending on the structure of the available panel data, several options will be possible. These options will differ in terms of the length of the time span and the number of periods per time span. In principle, comparisons can be made with the maximum overlap (e.g., $T_a[y_1, y_2, \ldots, y_{n-1}]$ vs. $T_b[y_2, y_3, \ldots, y_n]$), without any overlap (e.g., $T_a[y_1, y_2, \ldots, y_{n/2}]$ vs. $T_b[y_{n/2+1}, y_{n/2+2}, \ldots, y_n]$), or with something in between. Depending on these choices, poverty orderings may obviously differ. Thus, in addition to being robust to poverty indices, poverty lines, and aggregation procedures, one may also require poverty comparisons to be robust to the construction of the time spans. This aspect is illustrated empirically below.

3. Empirical Illustration

3.1. Data

To illustrate how stochastic dominance tests can be used to produce robust orderings of intertemporal poverty both over space and time, we use panel data for Indonesia and Peru.

For Indonesia, we use three waves of the Indonesian Family Life Survey (IFLS) conducted by RAND, the University of California Los Angeles, the University of Indonesia's Demographic Institute and the

Center for Population and Policy Studies of the University of Gadjah Mada in 1993 (IFLS1), 1997 (IFLS2), and 2000 (IFLS3). The IFLS is representative of 83% of the Indonesian population living in 13 of the nation's (at that time) 26 provinces. The IFLS is judged to be of very high quality because, among other things, individuals who have moved are tracked to their new location and, where possible, interviewed there (for details, see Strauss et al. [36]). Using the three waves, we built two panels, one from 1993 to 1997 and another from 1997 to 2000, each comprising roughly 32,000 individuals living in 7,000 households. We use real household expenditure per capita as the welfare measure, but refer to it as income in the following sections. Income is expressed in 1993 prices and adjusted by regional price deflators to the Jakarta price level.

For Peru, we use six waves (1997-2002) of the yearly Peruvian Encuesta Nacional de Hogares conducted by the Instituto Nacional de Estadística e Informática. The ENAHO is representative for the three rural and four urban areas of Peru. The 'panel-households' are only a sub-sample of all households interviewed. Each year, some households drop out of the panel and others are added (rotating panel). We construct several year-to-year panels, each containing, with a few exceptions, more than 5,000 individuals living in more than 1,000 households. Again, we use real household expenditure per capita as the income measure. Income is expressed in 2002 prices and adjusted by regional price deflators to the Lima price level.

To make income comparable between Indonesia and Peru, we convert local currencies to international 2002 $. Purchasing Power Parities (PPP) were taken from the Penn World Table 6.1 (see Heston et al. [24]).

3.2. Estimation and inference

To establish first order stochastic dominance empirically, it is sufficient–as shown by Duclos et al. [15]–to calculate the differences of

$\hat{F}(y_{1a}, y_{2a}, \ldots, y_{na})$ and $\hat{G}(y_{1b}, y_{2b}, \ldots, y_{nb})$ on a sufficiently narrow grid of test points and to test the statistical significance of these differences based on student t-tests (where "^" refers to estimated values). The relevant test domain changes based on the definition of poverty, i.e., whether union or intersection poverty is considered.

When applying the methodology presented in Section 2, one needs to define a maximum poverty set $\lambda^*(z_1, z_2, \ldots, z_n \in Z)$. Obviously, defining that frontier is always arbitrary. Again, we follow Duclos et al. [15] and estimate that frontier directly from our sample as the maximum λ^+ for which multiperiod poverty dominance holds. Then, we can locate within λ^+ all possible poverty frontiers for which, there is necessarily more poverty in time span a than in time span b. We can then judge on a case-by-case basis whether these critical sets and frontiers are wide enough to justify the conclusion on poverty dominance.

3.3. Robust multiperiod poverty comparisons for the two-period case

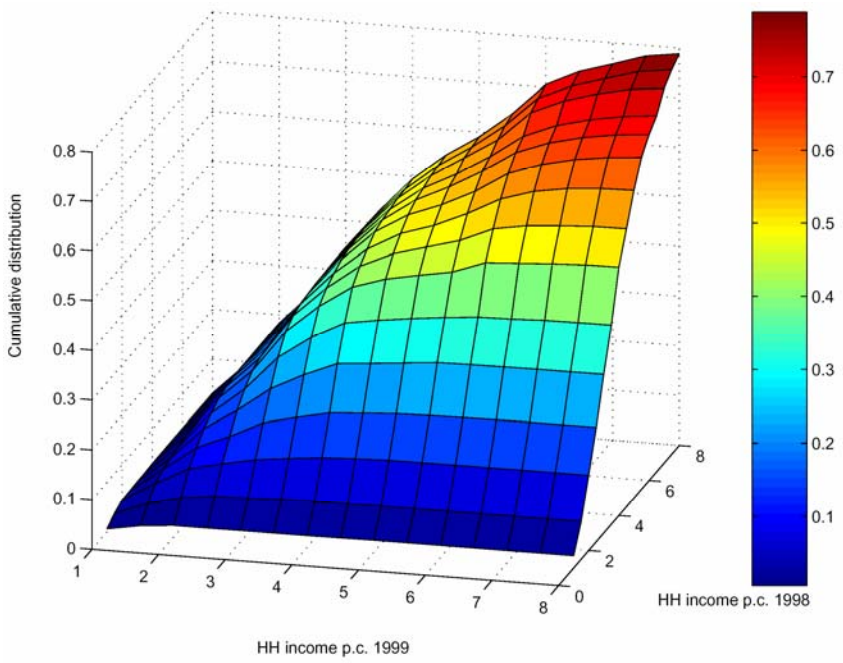
In this subsection, we first show empirically how to test for robustness to poverty lines. In this case, the arbitrary poverty line is assumed to be constant across the n-periods, i.e., within and across time spans. We then show how to test for robustness to the weighting (or aggregation) of periods within time spans. To keep the exposition simple and short, the empirical illustration primarily focuses on first order stochastic dominance tests by using the intersection definition of poverty. Again, the latter choice is made, because the focus is on chronic poverty.

3.3.1. Robustness to poverty lines

To analyze the robustness to the poverty line, we use three waves of the Peruvian household panel data and consider the time spans 1998 to 1999 and 1999 to 2000, i.e., we undertake a comparison of two subsequent

time spans. In terms of the chronic poverty literature, this means that we test whether one of both time spans systematically shows a higher degree of chronic poverty regardless of which common poverty line we apply in 1998, 1999, and 2000. Such comparisons of intertemporal, or multiperiod, poverty are still rare in the literature, but we anticipate that they will become increasingly common as more and more panel data becomes available.

According to Equation (9), FSD poverty comparisons can be made by testing for significant differences between the bisector of the dominance surface of 1998/99 and the bisector of the dominance surface of 1999/2000. Testing robustness to the poverty line implies testing all points on the bisector between income in period 1 and income in period 2. Again, given that we may not alter the poverty line across periods, we have to focus solely on the bisector. Figure 2 shows the dominance surface of the first time span 1998-1999. The x and y axes measure income per capita per day at the beginning (1998) and at the end (1999) of the time span. The third axis measures the cumulative share of individuals, who are below the points defined in the (x, y) domain.

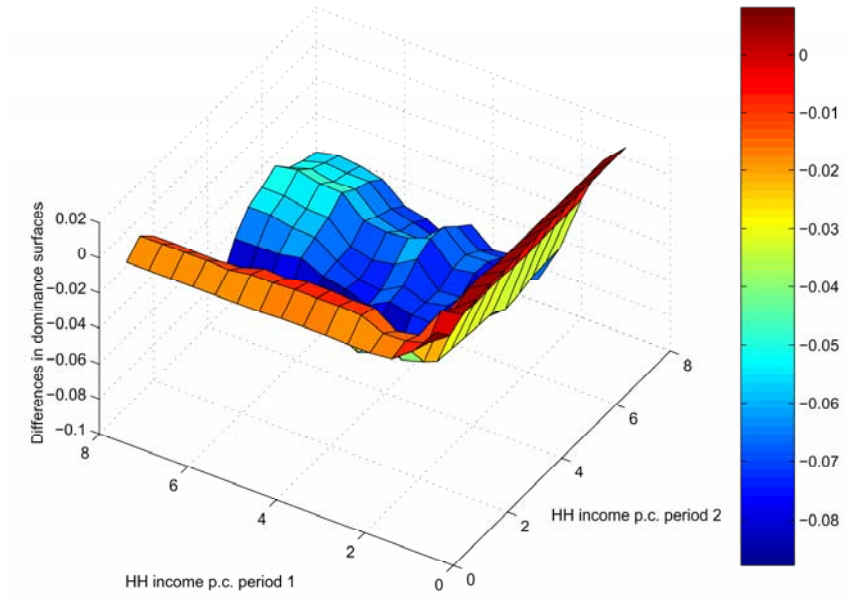


Income: Household income per capita per day in PPP Intl. $

Figure 2. Poverty in Peru: Dominance surface of the time span 1998/99.

Figure 3 shows the difference between the dominance surfaces of both time spans. Again, the relevant points can be found on the bisector of the graph, since we are testing only robustness to the poverty line (i.e., $z_1 = z_2$). The figure shows that, if we choose a relatively low poverty line (lower than 1 Intl. $), multiperiod poverty is higher in the first than in the second time span for all poverty indices belonging to the class $\Xi_{1,1}$. However, as we increase the poverty line, we first get insignificant differences and then, from a poverty line of 3 Intl. $ onward, we find that the cumulative share of people with an income below that poverty line increases faster for the second time span and, hence, multiperiod poverty becomes higher in that time span. In terms of

the chronic poverty literature, this result would mean that whether chronic poverty was higher over the period 1998 to 1999 or the period 1999 to 2000 depends on the poverty line we think is appropriate. For a poverty line of up to 1 Intl. $, we find that chronic poverty in Peru declined over time, because there were more individuals with less than $1 in 1998 and 1999 than in 1999 and 2000. For poverty lines between 1 Intl. $ and 3 Intl. $, we do not find a significant difference, i.e., chronic poverty in Peru did not change over time. For poverty lines above 3 Intl $, chronic poverty in Peru increased over time. This result holds, regardless of the poverty measure we use, as long as it belongs to the class $\Xi_{1,1}$. This result is also shown in Table 1.



Income: Household income per capita per day in PPP Intl. $

Figure 3. Poverty in Peru: Difference in dominance surfaces (1998/99 - 1999/2000).

Table 1. Poverty in Peru: Difference in dominance surfaces (1998/99 - 1999/2000)

Income period 1	**Income period 2**								
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
1.0	1								
1.5		0							
2.0			0						
2.5				0					
3.0					–1				
3.5						–1			
4.0							–1		
4.5								–1	
5.0									–1

Income: Household income per capita per day in PPP Intl. $;

1 indicates that the 1998/99 surface was significantly above the 1999/2000 surface,
–1 indicates the opposite, 0 indicates no significant difference.
Significance level: 5%.

The vertical axis in Table 1 shows income at the beginning of the time spans and the horizontal axis at the end of the time spans. The value '1' indicates a significant positive difference, i.e., 1999/2000 dominates 1998/99. '0' means an insignificant difference, while '–1' indicates a significant negative difference, i.e., 1998/99 dominates 1999/2000. We should of course check for poverty dominance at every possible point on this bisector, i.e., at every possible poverty line (e.g., $1, $1.01, $1.02, etc.). However, to keep the illustration simple, we report results only at those poverty lines that are multiples of $0.5.

Given that in this example *first order* stochastic dominance can only be established up to a poverty line of $1, it is useful to check whether higher order dominance can be established up to a higher poverty line. The cost is of course a reduction in the set of poverty measures for which

the dominance result holds. *Second order* stochastic dominance tests are shown in Table 2. It illustrates that second order poverty dominance holds up to the poverty line of $1.5 and, by definition, only for the class of measures $\Xi_{2,2}$.

Table 2. Poverty in Peru: Difference in second order stochastic dominance surfaces (1998/99 - 1999/2000)

Income period 1	**Income period 2**								
	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
1.0	1								
1.5		1							
2.0			0						
2.5				0					
3.0					0				
3.5						–1			
4.0							–1		
4.5								–1	
5.0									–1

Income: Household income per capita per day in PPP Intl. $;
1 indicates that the 1998/99 surface was significantly above the 1999/2000 surface,
–1 indicates the opposite, 0 indicates no significant difference.
Significance level: 5%.

3.3.2. Robustness to aggregation procedure

Now, we focus on the above-mentioned 'robustness to aggregation procedure', i.e., the weighting of the different periods that constitute a given time span. As we discussed above, there are many potential reasons for needing to reweight. The procedure is best illustrated by comparing multiperiod poverty across two socio-economic groups, where two observations over time are available for each group. The underlying question is: 'Is chronic poverty significantly higher in one of both groups regardless of the poverty measure used, regardless of the poverty line

used and regardless of the weight that is given to period 2 relative to period 1'?

We use the Indonesian data covering the time span 1993/1997 and compare employees in the formal private sector with self-employed individuals, mostly in the informal sector. The results are displayed in Table 3. We first focus robustness to the poverty line, i.e., we look at the bisector in Table 3. Since there are only few private sector employees with an income below $1 per person and day, the grid starts at the $1.25 poverty line. The findings show poverty dominance of private sector employees over the self-employed up to a maximum poverty line of $3.25, i.e., up to a poverty line of $3.25; there are significantly more self-employed individuals than employees in the formal private sector below the poverty line in 1993 and 1997.

Next, we check whether this result is also robust with respect to the aggregation procedure. Again, we do this by changing the poverty line in period 2 relative to the poverty line in period 1, i.e., we depart from the bisector in our grid. A higher poverty line in period 2 relative to period 1 means that in period 2 income receives a lower weight relative to income in period 1, i.e., a higher income is necessary to pass the poverty threshold. We find that the dominance result holds almost over the entire grid, except for the region around the $1.25 poverty line, as shown by the two '0's' in row 1. However, for any other poverty line between $1.5 and $3, poverty dominance holds regardless of the weighting scheme.

Table 3. Poverty in Indonesia: Differences in dominance surfaces (self-employed - private sector)

Income period 1	**Income period 2**											
	1.25	1.5	1.75	2.0	2.25	2.5	2.75	3.0	3.25	3.5	3.75	4.0
1.25	**1**	0	0	1	1	1	1	0	0	0	0	0
1.5	1	**1**	1	1	1	1	1	1	1	1	1	1
1.75	1	1	**1**	1	1	1	1	1	1	1	1	1
2.0	1	1	1	**1**	1	1	1	1	1	1	1	1
2.25	1	1	1	1	**1**	1	1	1	1	1	1	1
2.5	1	1	1	1	1	**1**	1	1	1	1	1	1
2.75	1	1	1	1	1	1	**1**	1	1	1	1	1
3.0	1	1	1	1	1	1	1	**1**	1	0	0	0
3.25	1	1	1	1	1	1	1	0	**1**	0	0	0
3.5	1	1	1	1	1	1	0	0	0	**0**	0	0
3.75	1	1	1	1	1	1	1	0	0	0	**0**	0
4.0	1	1	1	1	1	1	1	0	1	0	0	**0**

Income: Household income per capita per day in PPP Intl. $;

1 indicates that the 1993/97 surface of the self-employed was significantly above the 1993/1997 surface of the private sector employees, −1 indicates the opposite, 0 indicates no significant difference.

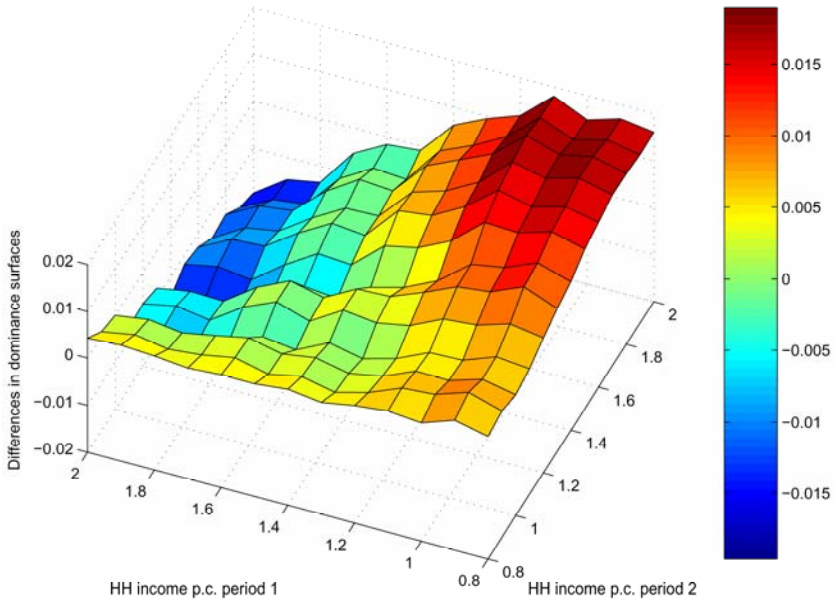
Significance level: 5%.

3.4. Robust multiperiod relative poverty comparisons for the two-period case within and across countries

To illustrate the concept of relative multiperiod poverty, we compare two time spans in Indonesia, namely, the time spans 1993/97 and 1997/2000. In terms of the chronic poverty literature, the comparison implies looking at changes in chronic poverty, where being chronically poor means to be persistently relatively poorer than the median person.

The difference in relative poverty between these two time spans is presented in Figure 4 (note that incomes are standardized to 50% of the median income, i.e., a value of 0.8 corresponds to 40% of the median). The

x and y axes measure relative income, $\tilde{y}$, at the beginning and the end of the time spans. The figure does not show any systematic pattern. This is confirmed by Table 4, which shows the corresponding grid of test points. Here, the 0 in the third row of the third column, for example, means that the share of individuals, who had less than 50% of the median income $(\tilde{y} = 1)$ in both periods did not significantly change between the time spans 1993/97 and 1997/2000. Hence, no clear-cut conclusions about changes in chronic relative poverty can be drawn.



Income: Household income per capita per day in PPP Intl. \$, standardized by a relative poverty line $\tilde{z} = 50\%$ of median income.

Figure 4. Relative poverty in Indonesia: Difference in dominance surfaces (1993/97 - 1997/2000).

Table 4. Relative poverty in Indonesia: Difference in dominance surfaces (1993/97 - 1997/2000)

Income **period 1**	***Income*** **period 2**												
	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.8	**1**	1	0	0	0	0	0	1	1	1	1	1	1
0.9	1	**0**	0	0	0	0	0	1	1	1	1	0	0
1.0	0	0	**0**	0	0	0	0	0	0	0	0	0	0
1.1	0	0	0	**0**	0	0	0	0	0	0	0	0	0
1.2	0	0	0	0	**0**	0	0	0	0	0	0	0	0
1.3	0	0	0	0	0	**0**	0	0	0	0	0	0	0
1.4	0	0	0	0	0	0	**0**	0	0	0	0	0	0
1.5	0	0	0	0	0	0	0	**0**	0	0	0	0	0
1.6	0	0	0	0	0	0	0	0	**0**	0	0	0	0
1.7	0	0	0	0	0	0	0	0	0	**0**	0	0	0
1.8	0	0	0	0	0	0	0	0	0	0	**0**	0	0
1.9	0	0	0	0	0	0	0	0	0	0	0	**0**	0
2.0	0	0	0	0	0	0	0	0	0	0	0	0	**0**

$\widetilde{Income}$ is household income per capita per day in PPP Intl. \$, standardized by a relative poverty line, $\tilde{z}$ = 50% of median income: $\widetilde{Income} = Income / \tilde{z}$.
1 indicates that the 1993/97 surface was significantly above the 1997/2000 surface, –1 indicates the opposite, 0 indicates no significant difference.
Significance level: 5%.

Our concept of relative poverty orderings is also applicable to cross-country comparisons. Absolute poverty comparisons using some agreed international poverty line are interesting, if countries have comparable and rather low living standards. But for countries with very different living standards or for richer countries, relative poverty might be more relevant. To illustrate this, we now compare Peru with Indonesia. Peru has a median income of 4.7 Intl. \$ and Indonesia 3.7 Intl. \$ per person per day. For both countries, we consider the time span 1997/2000.

Table 5 shows the grid of test points of differences in the two-period poverty surfaces ('Peru minus Indonesia'). Relative poverty is higher in Peru. Even though dominance cannot be established over the entire domain, the maximum poverty set for relative dynamic poverty is wide enough to conclude that there is dominance. The proportion of chronically relatively poor individuals is higher in Peru no matter what 'reasonable' relative poverty line or aggregation procedure is chosen. This seems plausible: Given the high inequality in Peru, also compared to Indonesia, we expect it to be harder to escape the bottom of the income distribution in Peru.

Table 5. Relative poverty in Peru and Indonesia: Difference in dominance surfaces Peru (1997/2000) - Indonesia (1997/2000)

Income **period 1**	***Income*** **period 2**												
	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
0.8	**1**	1	1	1	1	1	1	1	1	1	1	1	1
0.9	1	**1**	1	1	1	1	1	1	1	1	1	1	1
1.0	1	1	**1**	1	1	1	1	1	1	1	1	1	1
1.1	1	1	1	**1**	1	1	1	1	1	1	1	1	1
1.2	1	1	1	1	**1**	1	1	1	1	1	1	1	1
1.3	1	1	1	1	1	**1**	1	1	1	1	1	1	1
1.4	1	1	1	1	1	1	**1**	1	1	1	1	1	1
1.5	1	1	1	1	1	1	1	**1**	1	1	1	1	1
1.6	1	1	1	1	1	1	1	1	**1**	1	1	0	1
1.7	1	1	1	1	1	1	1	1	1	**0**	0	0	0
1.8	1	1	1	1	1	1	1	1	0	0	**0**	0	0
1.9	1	1	1	1	1	0	0	0	0	0	0	**0**	0
2.0	1	1	1	1	1	0	0	0	0	0	0	0	**0**

$\widetilde{Income}$ is household income per capita per day in PPP Intl. \$, standardized by a relative poverty line, $\tilde{z}$ = 50% of median income: $\widetilde{Income} = Income / \tilde{z}$.

1 indicates that the Peru surface was significantly above the Indonesia surface, –1 indicates the opposite, 0 indicates no significant difference.

Significance level: 5%.

3.5. Robust multiperiod poverty comparisons for the *n*-period case

The key questions addressed in this section relate to the construction of time spans in cases, where more than two rounds of panel data are available. How many periods should constitute a time span? Which period should be the end of the first and the beginning of the second time span?

To illustrate this problem, we use five waves of the Peruvian household panel data (1998-2002). For the sake of clarity, we require that, in each comparison, the first period of time span T_A be 1998 and the last period of time span T_B be 2002. We also abstain from making comparisons for different time span lengths. However, all remaining decisions regarding the construction of these time spans are arbitrary and, consequently, any poverty ordering may depend on exactly how the construction is carried out. We believe that there are at least five different comparisons that make sense from an economic point of view: three where we consider time spans comprising two periods, one where we consider time spans comprising three periods, and one where we consider time spans comprising four periods

[1998; 2000] vs. [2000; 2002]

[1998; 1999] vs. [2001; 2002]

[1998; 2001] vs. [1999; 2002]

[1998; 1999; 2000] vs. [2000; 2001; 2002]

[1998; 1999; 2000; 2001] vs. [1999; 2000; 2001; 2002]

Without imposing any further axioms on such comparisons, one may simply require that the assessment of changes of multiperiod, or chronic, poverty over time be robust to all possibilities mentioned above. Table 6 shows the results of such a dominance test. As one may expect, no significant ordering can be established in this case over the entire period 1998 to 2002. Using the $2 poverty line and comparing the time spans

[1998; 1999; 2000] and [2000; 2001; 2002], we can conclude that chronic poverty has fallen. However, taking the time spans [1998; 1999] and [2001; 2002] shows instead that no conclusion can be drawn. Hence, the poverty ordering depends not only on the chosen poverty line, but also on the way the time spans are constructed. Thus, unless dominance can be established for such cases, which is unlikely, further assumptions are necessary to reduce the number of cases for which dominance is desired.

Table 6. Poverty in Peru: Difference in dominance surfaces for several construction modes of time spans

Income	[98; 00] vs. [00; 02]	[98; 99] vs. [01; 02]	[98; 01] vs. [99; 02]	[98; 99; 00] vs. [00; 01; 02]	[98; 99; 00; 01] vs. [99; 00; 01; 02]
1	0	0	0	0	0
1.5	0	0	0	0	0
2	0	0	1	1	1
2.5	1	0	1	1	1
3	1	0	0	0	1
3.5	1	0	0	0	0
4	0	0	0	0	0
4.5	0	0	0	0	0
5	0	0	0	0	0

Income: Household income per capita per day in PPP Intl. $;

1 indicates that the earlier surface was significantly above the later surface,
–1 indicates the opposite, 0 indicates no significant difference.
Significance level: 5%.

4. Conclusion

In this paper, we suggested a straightforward way of conducting robust multiperiod poverty comparisons over time and space. Building on the multidimensional stochastic dominance methodology elaborated by

Duclos et al. [15], we created n-period income surfaces for different time spans. These surfaces were then ordered using dominance tests. Once dominance is established, the poverty ordering is robust to a wide range of poverty indices, to a wide range of poverty lines, and to a wide range of aggregation procedures. We also extended our framework to include the measurement of *relative* poverty. Finally, we highlighted some general problems with dynamic poverty comparisons, i.e., how time spans should be constructed, namely, which period should be the end of the first and the beginning of the second time span and how many periods should constitute a time span. We dealt with these questions by applying robustness tests with respect to these possibilities. All concepts were illustrated by using panel data for Peru and Indonesia. We conducted comparisons across time, groups, and countries.

What we discuss and propose is, on the one hand, a step back, because we (deliberately) avoid elaborating on the axioms that multiperiod poverty comparisons over time and space should satisfy and instead require robustness to a large set of possible assumptions. However, we consider this to be justified as some of the problems we highlight have received only little attention in the chronic poverty literature. Hence, highlighting these issues and illustrating the implications, will hopefully further enrich the debate on how to measure intertemporal poverty appropriately.

Acknowledgement

We are particularly grateful to Ravi Kanbur for the inspiring discussions with him and to Stefan Dercon, Jean-Yves Duclos and James Foster for their very useful comments and suggestions. We would also like to thank Jean-Yves Duclos and Abdelkarim Araar for having shared their Stata program for calculating stochastic dominance over multiple dimensions. All remaining errors are of course entirely our own. Finally, our thanks go to Philippe De Vreyer, Javier Herrera and Sandrine

Mesplé-Somps for having made available their Peruvian household data. Financial support for this research from the Chronic Poverty Research Center at the University of Manchester is gratefully acknowledged.

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